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Fracture initiation at a spherical inclusion in a matrix plastically deformed by shear

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The critical strain for fracture initiation of a metallic material with a spherical inclusion has been analysed using Eshelby's inclusion method for three types of fracture initiation models including the recovery effect by diffusion of atoms. When the elastic constant of inclusion approaches that of the matrix, the critical strain for fracture initiation becomes large in the case of uniform shear deformation of the matrix. It was found that the critical strain becomes large due to the diffusion of atoms, especially for inclusions of small size and a large elastic constant. The model in which the inclusion is cracked by the localized shear deformation can explain the observed inclusion size dependence of the strain for fracture initiation. The inclusion size dependence of the critical strain for fracture initiation by uniform shear deformation of the matrix is different from that by localized shear deformation. Therefore, it is important to know which mechanism governs the fracture.

1. Introduction

Fracture often initiates by cavitation at the interface between matrix and inclusion or by cracking of an inclusion when a ductile material containing a spherical inclusion is deformed plastically [1–7]. Several investigations have been made on the effects of volume fraction, particle spacing and size of inclusion on the plastic strain for fracture initiation [1–3, 6, 7].

The fracture initiation at an inclusion was discussed based on the dislocation theory [1, 8]. K. Tanaka *et al.* [9] and others [10, 11] investigated the fracture initiation at the interface of spherical, disc-like or needle-like inclusions under uniaxial tensile stress, using Eshelby's inclusion method. The results of analysis by K. Tanaka *et al.* [9] predicted that the fracture strain decreases with inclusion size, while the analysis based on the dislocation theory indicated the opposite inclusion size dependence of fracture strain [12]. Thus, the effect of inclusion size on fracture initiation is not well understood.

The plastic strain for fracture initiation is usually affected by the dynamic recovery. The dynamic recovery process can be controlled by plastic accommodation such as cross-slipping of dislocations [13], formation of prismatic loops [14, 15] or local lattice rotation [16] at relatively low temperature. The diffusion of atoms controls the dynamic recovery process around second phase particles at high temperatures above about $0.5T_m$ (T_m is the melting point of the matrix) [17]. The magnitude of the plastic strain for fracture initiation is one of the important factors in the hot working of metallic materials. However, there have been few works which treated a diffusional recovery effect on the fracture associated with an inclusion at high temperature [18–20].

In this study, the fracture initiation at a spherical inclusion in metallic materials was discusssed for three types of fracture models using Eshelby's inclusion method. An analysis was made to obtain the critical plastic strain for fracture initiation in the three cases where any recovery effect was not taken into account. The plastic strain for fracture initiation where diffusional recovery occurs was then calculated, using a micromechanics model which incorporated recovery effect by diffusion of atoms [19, 20]. Finally, a discussion based on the result of the present calculation was made on the effects of size and rigidity of inclusion on the plastic strain for fracture initiation at high temperature.

2. Analysis

2.1. Fracture strain under no recovery

2.1.1. Cavity initiation at the interface by uniform shear deformation

We consider that a shear stress, σ_{13}^A, is applied to an infinite material containing a spherical inclusion ($x_1^2 + x_2^2 + x_3^2 = a_1$) which is not deformed plastically, as shown in Fig. 1a. Uniform plastic deformation ($\varepsilon_{13}^{*\prime} = \varepsilon_{31}^{*\prime} = \gamma^*/2$ in the (x_1, x_2, x_3) coordinate) is caused in the matrix by the applied stress, σ_{13}^A. The internal stress state of this condition is identical with that when $-\varepsilon_{13}^{*\prime}(= -\varepsilon_{31}^{*\prime})$ occurs only in this inclusion [21]. The rigidity of inclusion, μ^*, is generally different from that of the matrix, μ. The actual shear stress in the inclusion is equal to the sum of two kinds of internal stresses, which are the stresses due to the inhomogeneity effect, $(\sigma_{13}^I)_{inh}$, and plastic deformation effect, $(\sigma_{13}^I)_{int}$. The magnitude of the maximum tensile stress, σ_{33}', in the (x_1', x_2', x_3') coordinate (Fig. 1b) is identical with this shear stress. According to K. Tanaka *et al.* [9], it is assumed that the stress criterion for cavity

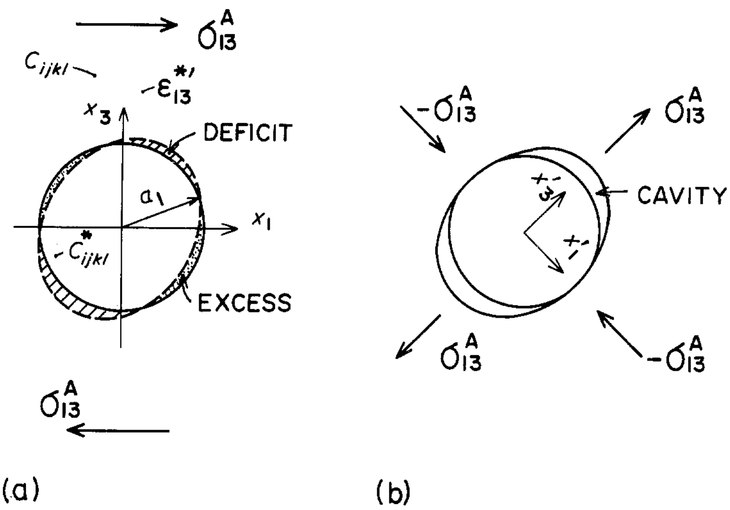

Figure 1 The initiation by uniform shear deformation of the matrix at the interface between inclusion and matrix. (a) Before cavitation; (b) after cavitation.

initiation is satisfied when the total tensile stress, σ'_{33}, reaches the theoretical strength of the interface,

$$\sigma'_{33} = (\sigma^{\mathrm{I}}_{13})_{\mathrm{inh}} + (\sigma^{\mathrm{I}}_{13})_{\mathrm{int}} \geqslant E/10 \text{ or } E^*/10 \quad (1)$$

Therefore, the critical strain for cavitation is

$$\gamma^* \geqslant (\gamma^*)_{\mathrm{sc}} = \frac{1}{A_1}\left[\frac{m'(1+v)}{5} - B_1\frac{\sigma^{\mathrm{A}}_{13}}{\mu}\right] \quad (2)$$

where, A_1 and B_1 are shape factors which are shown in the Appendix, and m' is a factor of rigidity, i.e. $m' = 1$ for $m < 1$ and $m' = m$ for $m > 1$, where $m = \mu^*/\mu$, and $v = v^*$ in the calculation.

The energy criterion for cavitation at the interface is satisfied when $G_1 > G_2$ [9], where G_1 and G_2 are Gibbs free energies before and after cavitation, respectively. G_1 is given by

$$G_1 = -\tfrac{1}{2}(\sigma^{\mathrm{I}}_{ij})_{\mathrm{int}}\,\varepsilon^{*\prime}_{ij}V - \sigma^{\mathrm{A}}_{ij}\varepsilon^{**}_{ij}V - \tfrac{1}{2}\sigma^{\mathrm{A}}_{ij}\varepsilon^{*}_{ij}V + S\gamma_{\mathrm{I-M}} \quad (3)$$

The first to fourth terms in Equation 3 are the elastic energy, the interaction energy, the energy change caused by inhomogeneity effect, and the interface energy [9]. V is the volume of an inclusion, and S is the surface area of an inclusion. Since the cavity is assumed to be caused by the maximum principal stress in the x'_3 direction, the total tensile stress in this direction becomes zero after cavitation at the interface [9]. Therefore, the Gibbs free energy after cavitation. G_2, is given by

$$G_2 = -\tfrac{1}{2}(\sigma^{\mathrm{I}}_{ij})'(\varepsilon^{*\prime}_{ij})'V - (\sigma^{\mathrm{A}}_{ij})(\varepsilon^{**}_{ij})'V - \tfrac{1}{2}(\sigma^{\mathrm{A}}_{ij})'(\varepsilon^{*}_{ij})'V + S(\gamma_{\mathrm{I}} + \gamma_{\mathrm{M}}) \quad (4)$$

where ( )′ indicates the quantity expressed by variables in the (x'_1, x'_2, x'_3) coordinate after cavitation. The fourth term is the sum of the surface energy of inclusion (γ_{I}) and matrix (γ_{M}). The values of $(\varepsilon^*_{ij})'$ and $(\varepsilon^{**}_{ij})'$ are shown in the Appendix. Then, the critical strain for cavitation which satisfies the energy criterion is obtained by

$$G_1 - G_2 = \tfrac{1}{2}(A_1 - A_2)\mu\gamma^{*2}V + (B_1 - B_2)\sigma^{\mathrm{A}}_{13}\gamma^*V + \tfrac{1}{2}(D_1 - D_2)\frac{\sigma^{\mathrm{A}}_{13}}{\mu}V + (\gamma_{\mathrm{I-M}} - \gamma_{\mathrm{I}} - \gamma_{\mathrm{M}})S > 0 \quad (5)$$

where the values of A_i, B_i and D_i are also shown in the Appendix. It is assumed in this study that $\gamma_{\mathrm{I}} = E^*a_0/10$, $\gamma_{\mathrm{M}} = Ea_0/10$, and $\gamma_{\mathrm{I-M}} = |\gamma_{\mathrm{I}} - \gamma_{\mathrm{M}}|$ [9], where a_0 is a lattice constant. Therefore, the critical fracture strain can be obtained by solving the quadratic equation of (Equation 5)

$$\gamma^* > (\gamma^*)_{\mathrm{ec}} = -\frac{15(1-v)}{7-5v}\frac{\sigma^{\mathrm{A}}_{13}}{\mu} + \left[\frac{12m'(1+v)a_0}{5(A_1 - A_2)}\right]^{\frac{1}{2}} \quad (6)$$

Cavitation is expected when both criteria given by Equation 2 and 6 are satisfied.

2.1.2. Internal cracking of an inclusion by uniform shear deformation

In this section, the stress and energy criteria are analysed for the internal cracking of an inclusion caused by uniform shear deformation of the matrix. As shown in Fig. 2a, the crack is caused by the

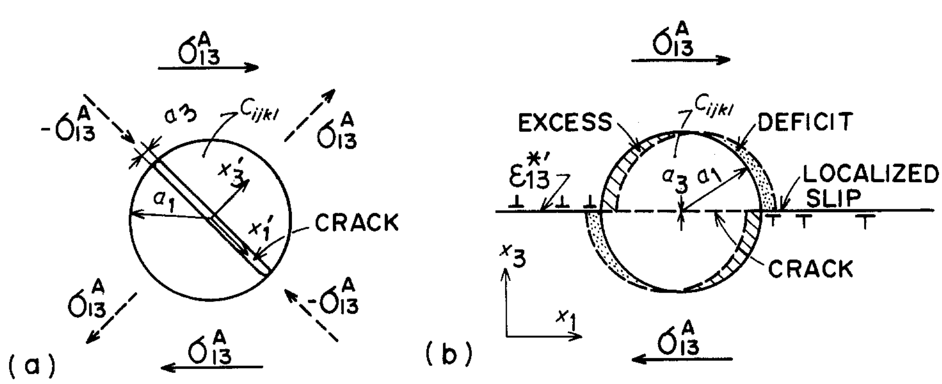

Figure 2 Internal fracture of an inclusion by shear deformation of the matrix. (a) Uniform shear deformation. (b) Localized shear deformation.

maximum tensile stress. The stress criterion given by Equation 1 should also be satisfied in this case. The solution of critical fracture strain, $(\gamma^*)_{sc}$, is identical with that expressed in Equation 2. If the elastic constant of the inclusion is the same as that of the matrix, the Gibbs free energy before cracking is given by

$$G_1 = -\tfrac{1}{2}(\sigma^1_{ij})'_i\varepsilon^{*\prime}_{ij}V - (\sigma^A_{ij})\varepsilon^{*\prime}_{ij}V \qquad (7)$$

where $(\sigma^1_{ij})'_i$ is the internal stress before cracking [21]. In this case

$$(\sigma^1_{11})'_i = (\sigma^1_{33})'_i = -2\mu\frac{7-5\nu}{15(1-\nu)}\varepsilon^{*\prime}_{13} \qquad (8)$$

and other stress components are zero, since the crack within the inclusion can be treated as an oblate spheroidal inclusion in which the total internal stress is zero. Therefore, the Gibbs free energy after cracking is obtained by

$$G_2 = -\tfrac{1}{2}(\sigma^1_{ij})'_i\varepsilon^{*\prime}_{ij}V - (\sigma^A_{ij})'\varepsilon^{*\prime}_{ij}V - \tfrac{1}{2}(\sigma^1_{ij})'_c\varepsilon^{*0}_{ij}V_c - (\sigma^A_{ij})'\varepsilon^{*0}_{ij}V - (\sigma^1_{ij})'_i\varepsilon^{*0}_{ij}V_c + S_c\gamma_1 \qquad (9)$$

where V_c and S_c are the volume and surface area of a crack, respectively. The first and second terms in Equation 9 are identical to those in Equation 7. The third to sixth terms are the elastic energy of an oblate spheroidal inclusion, the interaction energy between the internal stress and the applied stress, the interaction energy between the internal stresses of inclusion and crack, and the surface energy of the crack, respectively. The eigen strains in Equation 9 are shown in the Appendix. The critical strain of energy criterion for cracking, $(\gamma^*)_{ec}$, is given by $G_1 > G_2$. The energy criterion is satisfied, if

$$\gamma^* > (\gamma^*)_{ec} = -\frac{15(1-\nu)\sigma^A_{13}}{2(7-5\nu)\mu} + \left[\frac{135\pi(1-\nu)a_0}{8(7-5\nu)a_1}\right]^{\frac{1}{2}} \qquad (10)$$

2.1.3. *Internal cracking of localized shear deformation*

Internal cracking of an inclusion by localized slip [22] as shown in Fig. 2b is analysed in this section. We assumed that the slip band is not restrained at both ends. If there is no obstacle in the slip band, the internal stress in it is zero. When the elastic constant of the inclusion is the same as that of the matrix, the internal stress (σ^1_{13}), occurring by the shear strain $\varepsilon^{*\prime}_{13}$ in slip band is identical with the value that is presented by an oblate spheroidal inclusion with eigen strain $-\varepsilon^{*\prime}_{13}$ in the matrix. The critical strain of the stress criterion for cracking $(\gamma^*)_{sc}$, is given by $(\sigma^1_{13}) + \sigma^A_{13} \geqslant \mu/9$. The stress criterion is satisfied, if

$$\gamma^* \geqslant (\gamma^*)_{sc} = (\mu/9 - \sigma^A_{13})\frac{4(1-\nu)a_1}{(2-\nu)\pi a_3} \qquad (11)$$

The Gibbs free energy before cracking, G_1, is also expressed by Equation 7 but V should be replaced by the volume of the oblate spheroidal inclusion, V_0, in this case. The Gibbs free energy after cracking, G_2, is given by

$$G_2 = -\tfrac{1}{2}\sigma^A_{ij}\varepsilon^*_{ij}V + S_c\gamma_1 \qquad (12)$$

where

$$\varepsilon^*_{13} = \frac{2(1-\nu)a_1}{\pi(2-\nu)a_3}\frac{\sigma^A_{13}}{\mu} \qquad (13)$$

The energy criterion for cracking is satisfied when $G_1 > G_2$, and the corresponding strain is expressed by

$$\gamma^* > (\gamma^*)_{ec} = -\frac{4(1-\nu)a_1}{\pi(2-\nu)a_3}\frac{\sigma^A_{13}}{\mu} + \left[\frac{3(1-\nu)a_0a_1}{5\pi(2-\nu)a_3^2}\right]^{\frac{1}{2}} \qquad (14)$$

2.2. Fracture strain under diffusional recovery

The diffusion of atoms occurs around an inclusion at high temperature. The plastic deformation in the matrix results in an excess and a deficit of volume in the vicinity of an inclusion. Then, migration of atoms can occur from the region of excessive volume to that of deficit of volume [20]. In this study, the volume diffusion and grain boundary diffusion of atoms are considered as the recovery process. The excess (or deficit) of volume, ΔV, is given by $\gamma^* V/\pi$ [19] for the uniform shear deformation of the matrix (Fig. 1). γ^* is identical with the observed plastic strain, γ, when no recovery occurs. The measured critical strain for cavitation, γ_c, is defined corresponding to the critical strain, γ^*_c. The number of atoms, n, contained in ΔV is given by $\Delta V/\Omega$, where Ω is atomic volume. The driving force, F, for diffusion can be expressed by $F = \partial E_{el}/\partial n$, where E_{el} is the elastic strain energy described by the first term in Equation 3. Therefore, the flux of atoms is given by [20]

$$J = \frac{D}{kT\Omega}\operatorname{grad}F = \frac{\pi^2\mu A_1\Omega D}{kTa_1V}n \qquad (15)$$

where k is Boltzmann's constant, T is the absolute temperature, and D is the diffusion constant. Furthermore, the total cross-section of diffusion, S_d, and the diffusion distance for volume diffusion process are approximately πa_1^2 and a_1, respectively. The corresponding values for grain-boundary diffusion are $2\pi a_1\delta$ (δ is thickness of the interface and $\delta \simeq 2\boldsymbol{b}$, where $\boldsymbol{b}$ is the magnitude of Burgers vector) and a_1, respectively [20]. The total migration rate for diffusion is given by

$$\frac{dn}{dt} = \frac{dn_0}{dt} - S_dJ = \frac{dn_0}{dt} - C_sn \qquad (16)$$

where n_0 is the number of atoms produced by the applied strain γ and S_dJ is the number of migration of atoms by diffusion. If Equation 16 is solved by putting $n = 0$ for $t = 0$ under a constant rate, the solution is obtained as [11, 20]

$$\gamma^* = -\frac{\dot\gamma}{C_s}\left[1 - \exp\left(-\frac{C_s}{\dot\gamma}\gamma\right)\right] \qquad (17)$$

The cavitation can occur when γ^* reaches the critical strain which is given by Equations 2 and 6. The observed critical strain, γ_c, can be also obtained from Equation 17.

In the case of internal cracking of inclusion by

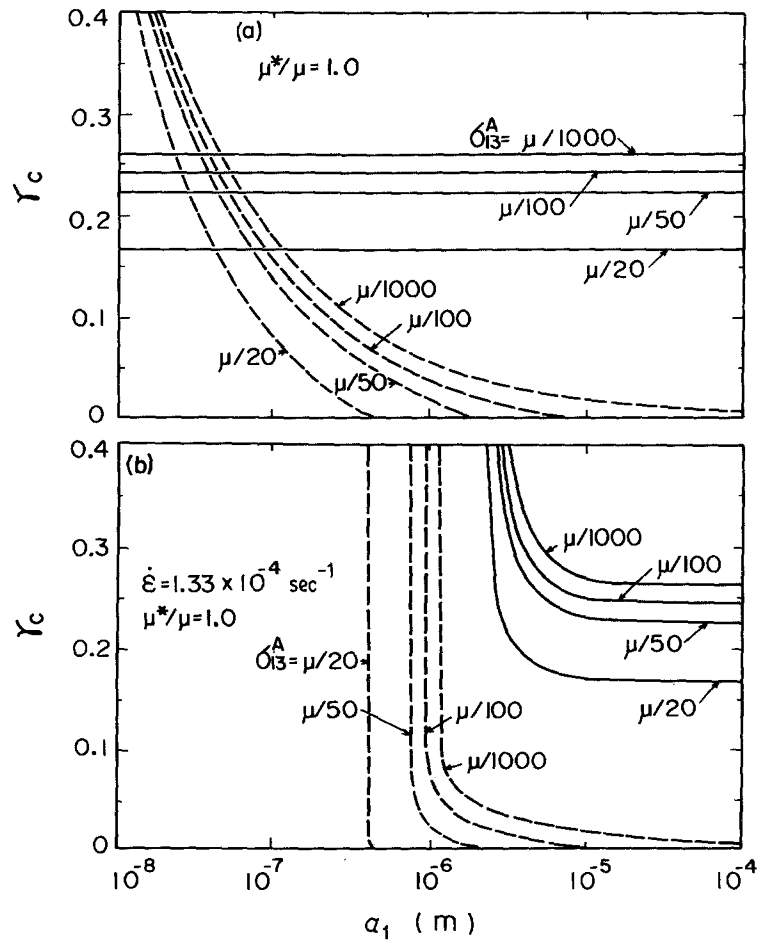

Figure 3 Relations between the critical strain for cavity initiation and the inclusion size by uniform shear deformation of the matrix. (a) No recovery, (b) recovery by volume diffusion. ——— Stress condition, --- energy condition.

uniform shear deformation, the effect of recovery can be taken into account by Equation 17. Shape factor A_1 is given by $(7 - 5\nu)/15(1 - \nu)$.

In the internal cracking by localized shear strain (Fig. 2b), the effect of recovery can be considered by the same procedure. In this case, $\Delta V = \gamma^* V$ [19] (V is the volume of an inclusion), S_d is approximately given by πa_1^2 for volume diffusion and $2\pi a_1 \delta$ for grain boundary diffusion. The diffusion distance is assumed to be a_1 for both cases, and the shape factor A_1 is given by $2\pi^2 \mu(2 - \nu) a_1 a_3^2/3(1 - \nu)$.

3. Numerical calculation and discussion

Fig. 3 shows the inclusion size dependence of the strain for cavity initiation by uniform shear strain. γ_c^* is identical with γ_c when no recoevery occurs. a_3 is assumed to be $(2.55 \times 10^{-10}\,\mathrm{m})$. As shown in Fig. 3a, when no recovery occurs, the critical strain of the stress criterion is constant, but that of the energy criterion is inversely proportional to the square root of the inclusion size. This tendency is identical with that appearing in the plastically deformed material with an inclusion under uniaxial tensile stress [9]. Fig. 3b shows the critical strain when the recovery by the volume diffusion of atoms is taken into account. The constants used in the calculation are, $D_v = 8.30 \times 10^{-18}\,\mathrm{m^2 sec^{-1}}$, $\Omega = 7.10 \times 10^{-6}\,\mathrm{m^3 mol^{-1}}$ and $\dot{\gamma} = 1.33 \times 10^{-4}\,\mathrm{sec^{-1}}$ [20]. The critical strain of the stress criterion abruptly increases with the decrease of inclusion size below about 10^{-5} m, and that of the energy criterion also increases with the inclusion size below about 10^{-6} m. This characteristic is caused by the stress relaxation due to the recovery, which occurs to a larger extent in a smaller inclusion. The characteristic of critical strain with the recovery of grain boundary diffusion was similar to that of volume diffusion.

Fig. 4 shows the effect of the difference in rigidity between inclusion (μ^*) and matrix (μ) on the critical strain for cavity initiation. The critical strain increases as the rigidity ratio ($m = \mu^*/\mu$) approaches unity. This tendency can be observed in both cases with and without recovery. It is interesting that the cavitation is relatively difficult to initiate in both cases when the rigidity of the inclusion approaches that of the matrix. The effect of the rigidity ratio on the critical strain is very large at high temperatures, especially where the ratio is greater than unity. This is attributed to the driving force of diffusional recovery being defined by the elastic strain energy (E_{el}) which is strongly influenced by the rigidity ratio.

Fig. 5 shows the critical strain for internal cracking by uniform shear strain of the matrix. The inclusion size dependence of the critical strain with or without recovery is the same as the corresponding result of calculation of cavity initiation in Fig. 4.

Fig. 6 shows the critical strain for internal cracking by localized shear strain. The critical strain for crack initiation without recovery increases with the inclusion size in the stress condition (Fig. 6a). It is also known from Fig. 6a that the crack initiation is governed by the stress criterion when the inclusion size becomes large. The critical strain of energy condition increases with the inclusion size for the relatively small inclusion size, but with further increase in inclusion size it reaches a maximum and then decreases abruptly. These tendencies are quite different from

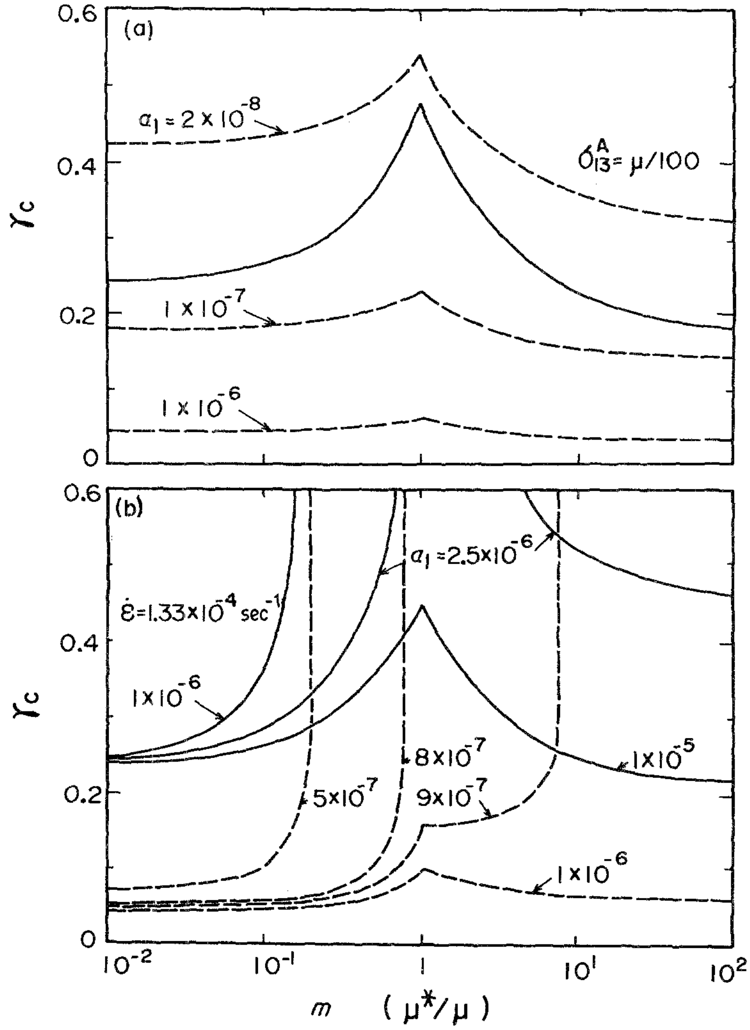

Figure 4 Relations between the critical strain for cavity initiation and the rigidity ratio ($m = \mu^*/\mu$) by uniform shear deformation of the matrix. (a) No recovery, (b) recovery by volume diffusion. ——— Stress condition, --- energy condition.

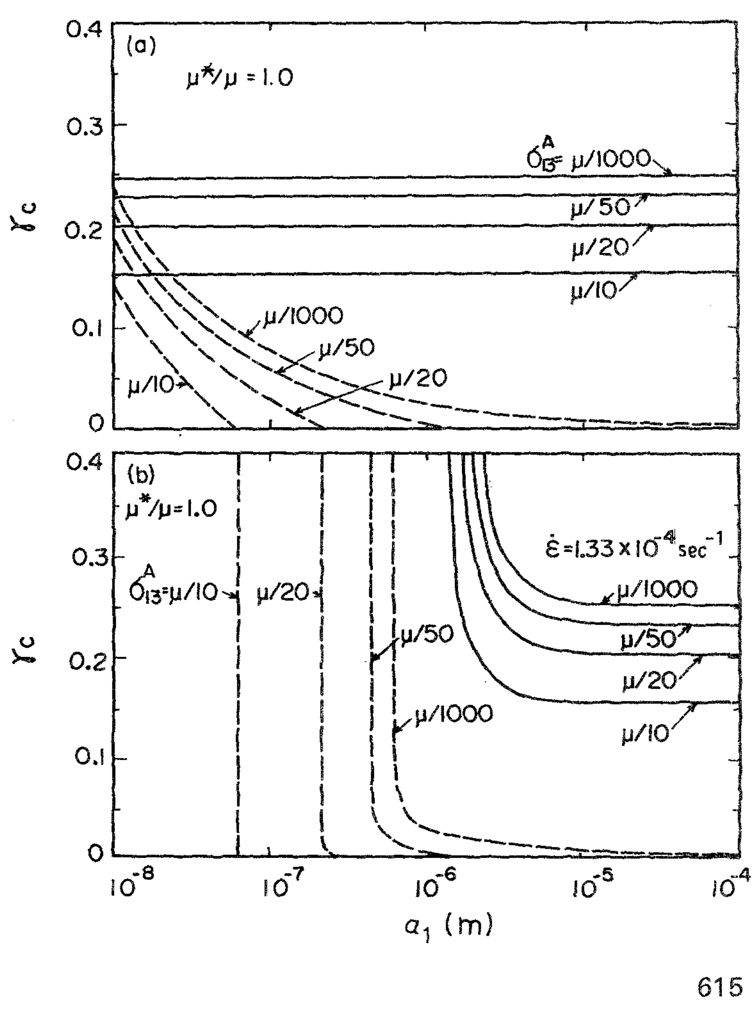

Figure 5 Relations between the critical strain for internal cracking and the inclusion size by uniform shear deformation of the matrix. (a) No recovery, (b) recovery by volume diffusion. ——— Stress condition, --- energy condition.

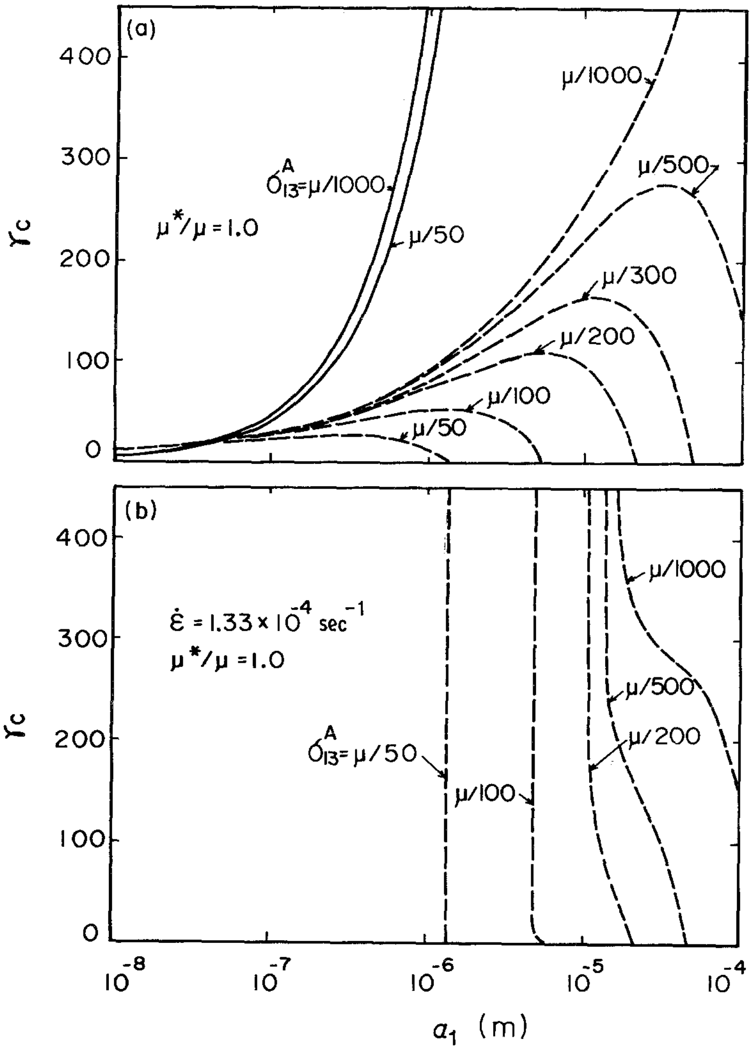

Figure 6 Relations between the critical strain for internal cracking and the inclusion size by localized shear deformation of the matrix. (a) No recovery, (b) recovery by volume diffusion. ——— Stress condition, --- energy condition.

those observed in other cases. At high temperatures, the critical strain of the stress criterion becomes too large to be exceeded owing to the recovery, while the energy criterion can be satisfied for a large inclusion size (Fig. 6b).

In this study, we treated the condition where the recovery is controlled by the diffusion of atoms. Therefore, these results should be recognized as the lower band of fracture initiation strain where the other recovery process also affects the fracture initiation strain.

It should be noted that the particle size dependence of critical strain which occurred by localized shear deformation is quite different from that observed in other cases. Fig. 7 shows the experimental results obtained by Inoue and Kinoshita [1], and the best fit curve of the analytical result by Goods and Brown [12] based on the dislocation theory. The particle size dependence of fracture initiation strain which was obtained by K. Tanaka *et al.* [9] using a model of cavitation by uniform tensile deformation, is different from the results shown in Fig. 7 [12]. It is found that the model of the internal cracking by localized shear strain (Section 2.1.3) gives the same tendency as the experimental results shown in Fig. 7, although the dynamic recovery such as plastic relaxation [14, 15] or lattice rotation [16] is not taken into account in this model. The observation of microstructure [1] indicated that the fracture initiation around the particle occurs relatively in the later stages of deformation, and that the internal cracking of the inclusion often initiates in the material containing the inclusion. Moreover, the internal cracking and localized deformation is reported elsewhere [3, 5, 23, 24]. The actual deformation includes both the localized and uniform strain components. Therefore, it is necessary for the discussion of fracture initiation to know what type of fracture occurs in the material considered.

4. Conclusion

The fracture initiation strain of the material containing a spherical inclusion was calculated using Eshelby's inclusion method for the uniform shear deformation and the localized shear deformation of the matrix. The results obtained are summarized as follows.

1. The effect of recovery by the diffusion of atoms is large when the inclusion size is small. However, the effect of recovery decreased abruptly with the increase of inclusion size and the fracture initiation strain approaches that of no recovery. This recovery effect is also remarkable when the rigidity of the inclusion is large compared with that of the matrix.

2. The strain for cavity initiation becomes large as the rigidity ratio of inclusion to matrix approaches unity. This implies that the cavitation is relatively difficult to initiate when the rigidity of the inclusion is almost the same as that of the matrix.

3. The model in which the inclusion is cracked by the localized shear deformation gives quite different

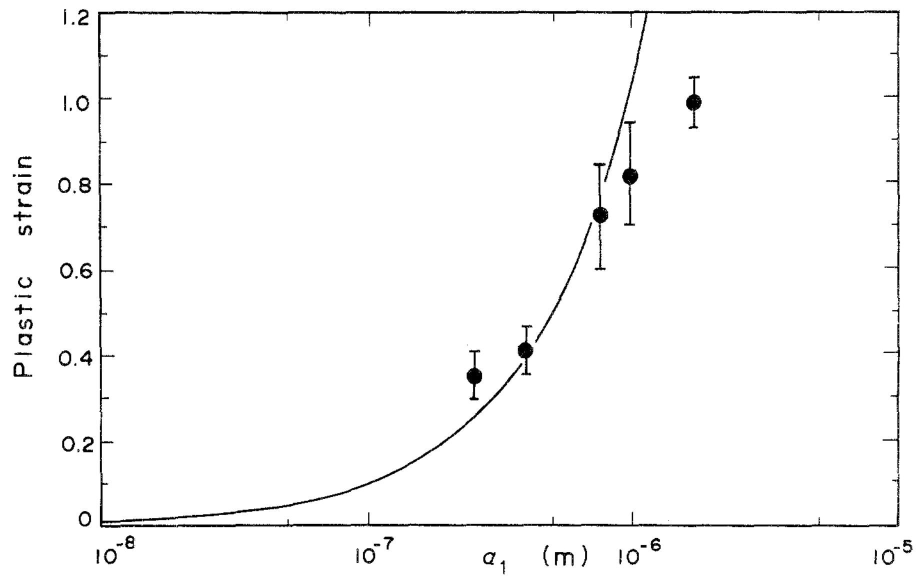

Figure 7 The dependence of inclusion size on the critical strain for fracture initiation. ⨡ From [1] (Fe–Fe_3C, room temperature); —— from [12].

results of the inclusion size dependence from that which occurs in the model of cavitation or cracking initiated by the uniform shear deformation. Therefore, it is important for the discussion of fracture initiation to know which mechanism governs the fracture initiation.

Appendix

For the fracture by uniform shear deformation, the shape factors before cavitation are given by

$$A_1 = \frac{(7-5\nu)\mu^*}{\mu(7-5\nu)+\mu^*(8-10\nu)}$$

$$B_1 = \frac{15(1-\nu)\mu}{\mu(7-5\nu)+\mu^*(8-10\nu)}$$

$$D_1 = \frac{15(\mu^*-\mu)(1-\nu)}{\mu(7-5\nu)+\mu^*(8-10\nu)}$$

and those after cavitation are

$$A_2 = \frac{(7-5\nu)[2\mu(7-5\nu)+\mu^*(1+\nu)(13-15\nu)]\mu^*}{4(1-\nu)[\mu(7-5\nu)+5\mu^*(1+\nu)][\mu(7-5\nu)+\mu^*(8-10\nu)]}$$

$$B_2 = \frac{15\mu^*\{2\mu(7-5\nu)+\mu^*(1+\nu)(13-15\nu)\}}{4[\mu(7-5\nu)+\mu^*(8-10\nu)][\mu(7-5\nu)+5\mu^*(1+\nu)]}$$

$$D_2 = \frac{15(1-\nu)[5\mu^{*2}(1+\nu)-4\mu^2(7-5\nu)-2\mu\mu^*(11-10\nu)]}{4[\mu(7-5\nu)+5\mu^*(1+\nu)][\mu(7-5\nu)+\mu^*(8-10\nu)]}$$

The eigen strains of equivalent inclusion after cavitation are given by

$$(\varepsilon_{22}^*)' = \frac{-15(1-\nu)(\mu-\mu^*)\{2(7-5\nu)\mu+(13-5\nu)\mu^*\}\sigma_{13}^{A}}{4\mu[(7-5\nu)\mu+2(4-5\nu)\mu^*][(7-5\nu)\mu+5(1+\nu)\mu^*]}$$

$$(\varepsilon_{22}^*)' = \frac{-45(1-\nu)(1-5\nu)(\mu-\mu^*)\mu^*\sigma_{13}^{A}}{4\mu[(7-5\nu)\mu+2(4-5\nu)\mu^*][(7-5\nu)\mu+5(1+\nu)\mu^*]}$$

$$(\varepsilon_{33}^*)' = \frac{15(1-\nu)(2\mu+\mu^*)\sigma_{13}^{A}}{4\mu[(7-5\nu)\mu+5(1+\nu)\mu^*]}$$

$$(\varepsilon_{11}^{**})' = \frac{3\mu^*[(57-40\nu-25\nu^2)\mu^*+(7-5\nu)(9-5\nu)\mu]\gamma^*}{4[(7-5\nu)\mu+2(4-5\nu)\mu^*][(7-5\nu)\mu+5(1+\nu)\mu^*]}$$

$$(\varepsilon_{22}^{**})' = \frac{3\mu^*(7-5\nu)(1-5\nu)(\mu^*-\mu)\gamma^*}{4[(7-5\nu)\mu+2(4-5\nu)\mu^*][(7-5\nu)\mu+5(1+\nu)\mu^*]}$$

$$(\varepsilon_{33}^{**})' = \frac{-3\mu^*(1+5\nu)\gamma^*}{4[(7-5\nu)\mu+5(1+\nu)\mu^*]}$$

In the internal cracking by uniform shear strain, the eigen strains in G_2 (Equation 9) are given by

$$\varepsilon_{11}^{*0} = -\left(\frac{3-2\nu}{4}\right)\frac{(\sigma_{11}^{I})_c'}{\mu}$$

$$\varepsilon_{22}^{*0} = -\left(\frac{1-2\nu}{4}\right)\frac{(\sigma_{11}^{I})_c'}{\mu}$$

$$\varepsilon_{33}^{*0} = \left[2(1-\nu)\frac{a_1}{\pi a_3}-\frac{11-12\nu}{8}\right]\frac{(\sigma_{11}^{I})_c'}{\mu}$$

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